Estimation-Based Synthesis of H_∞-Optimal Adaptive Equalizers over Wireless Channels

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Abstract

This paper presents a systematic synthesis procedure for H_∞-optimal adaptive FIR equalizers over a time-varying wireless channel. The channel is assumed to be frequency selective with Rayleigh fading. The proposed equalizer structure consists of the series connection of an adaptive FIR filter and a fixed equalizer (designed for the nominal channel). Adaptation of the weight vector of the adaptive FIR filter is achieved using the H_∞-optimal solution of an estimation-based interpretation of the channel equalization problem. Due to its H_∞-optimality, the proposed solution is robust to exogenous disturbances, and enables fast adaptation (i.e., a short training period) without compromising steady-state performance of the equalization. Preliminary simulation are presented to support the above claims.

1 Introduction

Although the problem of channel equalization has been extensively studied in the literature (in light of the s-hear volume of the literature on this subject we will just refer to [1]) the growth of wireless communications has presented new challenges. In particular, the time-varying nature of the channels (due to multipath and doppler effects) has encouraged new research on equalization techniques([2], [3], [4], [5], [6]).

In this paper we suggest a systematic design procedure to the adaptive equalization problem for time-varying channels. We will throughout assume that the channel is a frequency selective Rayleigh fading channel, and that training symbols are sent with each data block (due to the time varying nature of the channel) to train the receiver equalizer.

The proposed equalizer consists of two portions, a nominal and an adaptive one. The nominal equalizer is an adaptive equalizer which is trained once in the beginning to capture the nominal characteristics of the channel (and in the absence of the adaptive portion). Future training is then done in the presence of both filters. However, the nominal portion is kept fixed, while the adaptive portion is trained.

Subsequent training of the adaptive portion uses an H_∞ criterion, and so the proposed adaptive equalizer is robust to exogenous disturbances. Moreover, it enables fast adaptation (i.e., it only requires a short training period) without compromising the steady-state performance of the equalizer. Preliminary simulations indicate the feasibility of the algorithm, and demonstrate superior performance, with shorter training periods, over LMS-based adaptive algorithms.

The approach is applicable (due to its systematic nature) to both FIR and IIR adaptive filter design, for the nominal as well as for the adaptive portions, even though this paper only considers the FIR case.

We also note that the classical adaptive filtering solution to adaptation of an adaptive filter in series connection with the nominal equalizer(see Fig(1) is filtered-X LMS ([7],[8],[9],[10],[11]) . The approach in this paper however is based on the EBAF algorithm [8][12].

Moreover the derivations in this paper assume vector-valued signals, which readily extend the scope of the results to matrix-valued channels (essentially multiple-input/multiple-output systems). It must be noted that although the problem of MIMO equalization has been readily studied in the literature ([13], [14]), none of the methods developed so far have been *adaptive*. The

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adaptive spatio-temporal equalization techniques such as those proposed in [15], though adaptive, are not MI-MO channel equalizers.

In contrast to classical adaptive algorithms (e.g. FxLM-S), for the approach presented in this paper the synthesis of single-channel and multi-channel adaptive algorithms are virtually identical. This similarity is a direct result of the way the synthesis problem is formulated (see Section 3). However the subject of adaptive MI-MO equalization is the subject of another paper([16]) and will not be addressed in this paper.

This paper is organized as follows. In section 2 the main concepts of the proposed estimation-based-adaptive-filtering (EBAF) algorithm are discussed. The EBAF problem formulation is developed in section 3. In section 5 the implementation scheme for EBAF Equalization is described. Section 6 provides some simulation results. We finally conclude with section 7.

2 EBAF Algorithm - Main Concepts

The principal goal of this section is to introduce the underlying concepts of the new estimation based adaptive filtering (EBAF) ([8],[17]) algorithm which is the basis for the EBAF equalizer proposed in this paper. It also lays the foundation for mathematical formulation of the algorithm to which Section 3 is devoted.

Referring to Fig. 1, the objective in this adaptive filtering problem is to adapt the weight vector of the adaptive FIR equalizer such that the output of the nominal equalizer, $y(k)$, is in some measure (to be specified later) close to the training sequence $u(k)$.

Equivalently, given the available measurement at the output of the communication channel, $d(k)$, and the *known* structure of the nominal equalizer[1], we would like to produce an *optimal* estimate of the training sequence, $u(k)$, given the existing structure of the EBAF equalizer (i.e. the series connection of the adaptive FIR filter and the nominal equalizer). The optimality criterion will be introduced in the next section. To do so, we first introduce a state space representation for the EBAF equalizer (Figure 2). In this model, the weight vector of the adaptive FIR filter will be treated as a portion of the overall state variable. We then formulate a standard estimation problem, the solution of which provides estimates of the training sequence, $u(k)$, as well as 'estimates' of the *optimal* weight vector. This 'optimal' weight vector will minimize (in some appropriate measure) the error between the output of the EBAF equalizer and the training sequence (i.e. $y(k) - u(k)$). The solution of the estimation problem will be used as the adaptation criterion for the weight vector in the adaptive FIR filter (see Section 5).

[1] We will explain how to construct the nominal equalizer shortly

The following Section presents the mathematical formulation for the estimation problem.

3 Problem Formulation

Figure 2 shows a block diagram representation of the EBAF equalizer. We assume a state space model, $[\ A,\ B,\ C,\ D\ ]$, for the nominal (fixed) equalizer. The nominal equalizer is designed (using any common equalization technique) for the 'nominal' channel. Note that this design is done without the presence of the adaptive FIR filter. In other words, the nominal equalizer is the 'best' (with respect to an appropriate criterion) equalizer for the nominal communication channel if the channel was time invariant. The adaptive FIR equalizer in the proposed EBAF equalizer, then accounts for the variations of the channel over time. The exogenous disturbance $\mathcal{V}_m(k)$ is included to account for the effect of the AWGN (at the output of the channel), corrupting the reference signal, d(k).

We treat the weight vector, $W(k) = [\ w_0(k)\ w_1(k)\ \cdots\ w_N(k)\ ]^T$, as the state vector capturing the trivial dynamics, $W(k+1) = W(k)$, that we assume for the FIR filter. $\xi^T = (\ W^T(k) \quad \theta^T(k)\ )$ is then the state vector for the overall system (where $\theta(k)$ captures the dynamics of the nominal filter in this model).

The state space representation of the system is then

$$\overbrace{\begin{bmatrix} W(k+1) \\ \theta(k+1) \end{bmatrix}}^{\xi_{k+1}} = \overbrace{\begin{bmatrix} I_{(N+1)\times(N+1)} & 0 \\ Bh^*(k) & A \end{bmatrix}}^{F_k} \overbrace{\begin{bmatrix} W(k) \\ \theta(k) \end{bmatrix}}^{\xi_k} \quad (1)$$

where $h(k) = [d(k)\ d(k-1)\ \cdots\ d(k-N)]^T$ captures the effect of the reference input $d(\cdot)$. For this system, the 'measured' output is $m(k) = y(k) + e(k)$ (this quantity is used in section 4 and is defined in [17]). In terms of the system parameters the 'measured' output can be expressed as

$$m(k) = \overbrace{\begin{bmatrix} Dh^*(k) & C \end{bmatrix}}^{H_k} \begin{bmatrix} W(k) \\ \theta(k) \end{bmatrix} + \mathcal{V}_m(k) \quad (2)$$

We also define $s(k)$, the estimated quantity, to be $u(k)$. The end goal of the estimation based approach is to set the weight vector in the adaptive FIR equalizer such

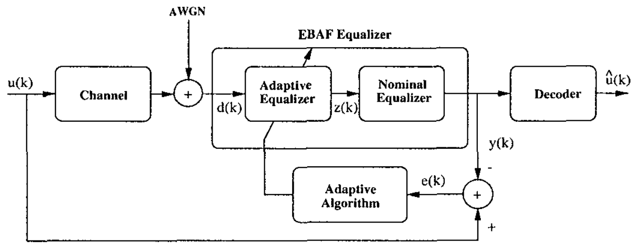

Fig. 1: Block diagram of the System

that the output of the nominal equalizer, $y(k)$ in Fig. 1, best matches $u(k)$. Therefore, $u(k)$ should be the estimated quantity. $s(k)$ then can be expressed in terms of system parameters as

$$s(k) = \overbrace{\begin{bmatrix} Dh^*(k) & C \end{bmatrix}}^{L_k} \begin{bmatrix} W(k) \\ \theta(k) \end{bmatrix} \qquad (3)$$

Note that $m(k) \in \mathcal{R}^{1\times 1}$, $s(k) \in \mathcal{R}^{1\times 1}$, $\theta(k) \in \mathcal{R}^{N_s\times 1}$, and $W(k) \in \mathcal{R}^{(N+1)\times 1}$. All matrices are then of appropriate dimensions.

Any estimation algorithm can now be used to generate $\hat{s}(k|k) \triangleq \mathcal{F}(m(0), \cdots, m(k))$ (a causal estimate of the desired quantity, $s(k)$) such that some *closeness* criterion is met. We consider an H_∞ optimal estimation algorithm here.

The main objective is to limit the worst case energy gain from the measurement disturbance and the initial condition uncertainty to the error in a causal estimate of $s(k)$. In other words, it is desired to find an H_∞ suboptimal causal estimator $\hat{s}(k|k) = \mathcal{F}(m(0), \cdots, m(k))$ such that

$$\sup_{\mathcal{V}_m,\ \xi_0} \frac{\sum_{k=0}^{M} [s(k) - \hat{s}(k|k)]^* [s(k) - \hat{s}(k|k)]}{(\xi_0 - \hat{\xi}_0)^* \Pi_0^{-1} (\xi_0 - \hat{\xi}_0) + \sum_{k=0}^{M} \mathcal{V}_m^*(k) \mathcal{V}_m(k)} \le \gamma^2 \qquad (4)$$

for a given scalar $\gamma > 0$. The question of optimality of the solution is then answered by finding the *infimum* value among all feasible γs. Here Π_0 is a positive-definite matrix. Note that, in this case there is no statistical assumption regarding the measurement disturbance.

4 H_∞-Optimal Solution

The optimal value of γ is shown to be 1, for which the Riccati recursion reduces to a simple Lyapunov recursion. For details please refer to [8]. The following theorem summarizes the simplified solution to the estimation problem.

4.1 γ-Suboptimal Finite Horizon Filtering Solution

The following theorem summarizes the simplified solution to the estimation problem.

Theorem: *For the system described by Equations (1)-(3), the central H_∞-optimal estimator (for $\gamma_{opt} = 1$) is given by*

$$\hat{\xi}_{k+1} = F_k \hat{\xi}_k + K_{1,k} \left(m(k) - H_k \hat{\xi}_k \right), \quad \hat{\xi}_0 = 0 \qquad (5)$$

$$\hat{s}(k|k) = L_k \hat{\xi}_k + \left(L_k P_k H_k^* \right) R_{He,k}^{-1} \left(m(k) - H_k \hat{\xi}_k \right) \qquad (6)$$

with $K_{1,k} = (F_k P_k H_k^*) R_{He,k}^{-1}$ and $R_{He,k} = I_p + H_k P_k H_k^*$, where P_k satisfies the Lyapunov recursion

$$P_{k+1} = F_k P_k F_k^*, \quad P_0 = \Pi_0. \qquad (7)$$

Main features of the algorithm are discussed in [8],[17]. Note that when $[A,\ B,\ C,\ D] = [0,\ 0,\ 0,\ I]$, (i.e. the output of the adaptive FIR equalizer directly matches $u(k)$ in Figure 1), then the results we have derived so far will reduce to the simple LMS algorithm [18].

5 Implementation Scheme for EBAF Equalization

Now, we can outline the implementation algorithm as follows:

1. Start with $\hat{W}(0) = 0$, $\hat{\theta}(0) = 0$ as estimator's initial guess for the state vector in EBAF equalizer. Also, assume that $u(0)$ is the initial value of the training sequence.
2. For $0 \le k \le M$ (where M is the number of training sequence):
   (a) Form signal $z(k) = h_k^* \hat{W}(k)$,
   (b) Apply $z(k)$ to the nominal equalizer,
   (c) Form the measurement, $m(k) = e(k) + y(k)$,
   (d) Use the H_∞-optimal estimator's state update, Eqs. (5), to find the H_∞-optimal estimate of

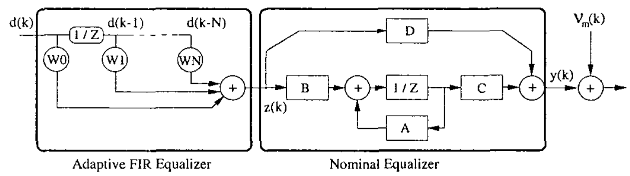

Fig. 2: Block diagram for the EBAF equalizer

the weight vector in the adaptive FIR equalizer (i.e. $\hat{W}(k+1)$). Note that $\hat{\theta}(k+1)$ should also be stored for the next estimation update.

(e) Propagate the Riccati matrix P_k using Eq. (7).

3. Go to 2.

6 Simulation Results

The concepts presented in the previous sections are tested through simulations.

The block diagram of the system is as shown in Figure 1. We consider baseband representation of the signals and system. The input signal, passes through the Raleigh fading channel and then is affected by additive white Gaussian noise (AWGN). At the receiver the received signal is passed first through an equalizer and then through the decoder. An uncoded BPSK signal constellation is used for the transmitted data.

The channel is modeled as a two ray Raleigh [19]; which considers the impulse response to be two delta functions, which have independent fades, and have a time delay of one symbol period. This is sufficient time delay to induce frequency selective fading upon the input signal.

We consider two adaptive equalization schemes. One is the proposed EBAF method, which consists of the series combination of an adaptive FIR filter and a fixed nominal equalizer. As mentioned earlier, the nominal equalizer is trained once at the beginning, and in subsequent trainings remains fixed while the adaptive portion is trained. The second method uses a linear transversal filter with fixed length, that is trained using the LMS algorithm.

In the LMS case, in order to capture the information from previous trainings, the weight vector of previous training period is used as initial weight vector during current training period. To make a fair comparison, the length of the LMS-based equalizer is chosen to be equal to the length of the adaptive portion of the EBAF equalizer. We have used 16 taps as the maximum length in both cases.

Figure 6 shows the convergence characteristics of the adaptive filters(EBAF and LMS) during the training period, for four different time instances. The plots are shown for a high SNR case ($SNR = 30dB$).

As seen from the figure, for a given final mean squared error (MSE), the EBAF method is an order of magnitude faster than LMS. This is due the fact that previous channel state information are incorporated into the fixed nominal filter, which results in a faster adaptation of the adaptive portion.

In order to provide a more general performance measure for the proposed technique we have plotted the bit error rate versus SNR in Figure 4 for the above two cases. These plots were obtained using Monte-Carlo trials (10 million runs per SNR point) using 1dB SNR increment steps. As mentioned earlier no error-correcting codes have been used.

The adaptive equalizers in both cases, adapt to the channel changes, using transmitted training sequences sent with each block of data. For the simulations we have assumed the channel to be block time invariant (the channel remains fixed during the training period). The training period of the LMS algorithm has been chosen to be five times longer since the algorithm takes longer to converge.

Our Simulation results, so far indicate the feasibility of the algorithm, as well as fast adaptation (i.e., a short training period) without compromising steady-state performance of the equalization.

7 Conclusion

A systematic synthesis procedure for H_∞-optimal adaptive FIR equalizers for a time varying wireless has been proposed. The channel is assumed to be frequency selective with Raleigh fading.

The proposed adaptive equalizer is robust to exoge-

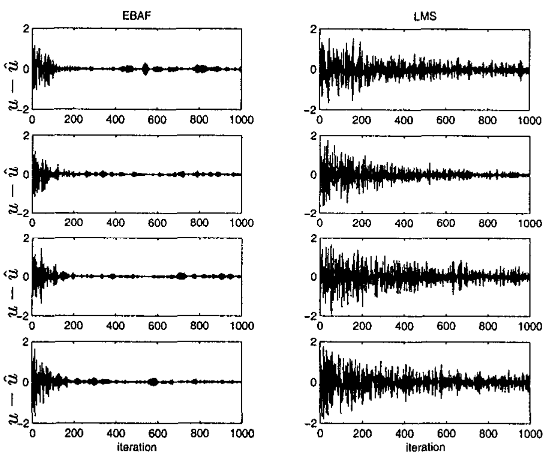

Figure 3: Convergence characteristics of the adaptive filters (EBAF and LMS)

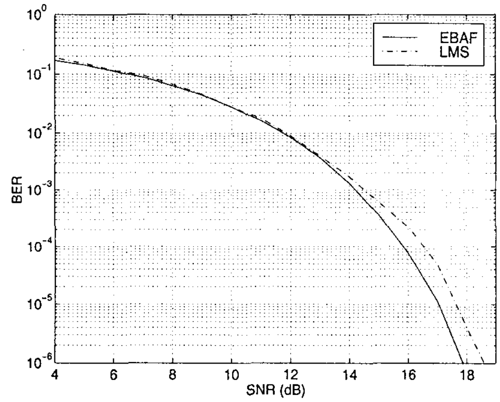

Figure 4: Bit Error Rate (BER) versus SNR.

nous disturbances, and enables fast adaptation (i.e. short training period) without compromising steady-state performance of the equalization. This is due to the H_∞ criterion chosen to adapt the weight vector.

With the growing complexity of the wireless communication channels the equalization of MIMO channels requires sophisticated equalization algorithms with guaranteed stability and predictable performance. The systematic approach presented in this paper is a first step towards solving this more general problem in a convincing fashion.

Our Simulation results, so far indicate the feasibility of the algorithm, as well as better performance with shorter training period.

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